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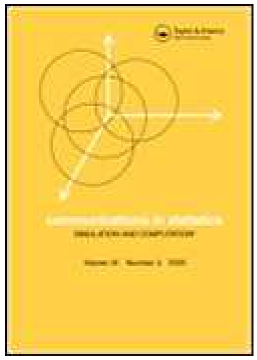

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Journal:	*Communications in Statistics - Simulation and Computation*
Manuscript ID:	LSSP-2009-0054
Manuscript Type:	Original Paper
Date Submitted by the Author:	27-Feb-2009
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A Synthetic Scaled Weighted Variance Control Chart for Monitoring the Process Mean of Skewed Populations

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Abstract

In this paper, a synthetic scaled weighted variance $\bar{X}$ (synthetic SWV-$\bar{X}$) control chart is proposed to monitor the process mean of skewed populations. This control chart is an improvement over the synthetic weighted variance $\bar{X}$ (synthetic WV-$\bar{X}$) chart suggested by Khoo et al. in 2008, in the detection of a negative shift in the mean. A comparison between the performances of the synthetic SWV-$\bar{X}$ and synthetic WV-$\bar{X}$ charts are made in terms of the average run length (ARL) values for the various levels of skewnesses as well as different magnitudes of positive and negative shifts in the mean. A method to construct the synthetic SWV-$\bar{X}$ chart is explained in detail. An illustrative example is also given to show the implementation of the synthetic SWV-$\bar{X}$ chart.

Keywords: Synthetic $\bar{X}$ chart; Weighted Variance $\bar{X}$ chart; Scaled; Weighted Variance $\bar{X}$ chart; Johnson distributions.

1 Introduction

The synthetic control chart was introduced by Wu & Spedding (2000b) as an improvement over the Shewhart $\bar{X}$ chart for detecting shifts in the mean of a normally distributed process. The synthetic chart for the mean integrates the Shewhart $\bar{X}$ chart and the conforming run length (CRL) chart. Wu & Spedding (2000b) showed that for moderate shifts in the mean, the synthetic chart

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reduces the out-of-control average run length (ARL) by nearly half while maintaining the same in-control ARL. They also demonstrated that the synthetic chart outperforms the exponentially weighted moving average (EWMA) chart and the joint $\bar{X}$-EWMA charts when the mean shift is greater than 0.8σ.

Other works on synthetic charts are as follow : Wu & Spedding (2000a) presented a program in C to design a synthetic chart that minimizes the out-of-control ARL based on an optimization model. Wu & Yeo (2001) and Wu, Yeo & Spedding (2001) proposed synthetic charts for detecting increases in the fraction nonconforming. Calzada & Scariano (2001) found that the synthetic chart of Wu & Spedding (2000b) is reasonably close to the normal theory values for moderate nonnormality or when the sample size n is large. Davis & Woodall (2002) presented a Markov chain model of the synthetic chart suggested by Wu & Spedding (2000a) and used it to evaluate the chart's zero-state and steady-state ARL performances, besides altering the chart to achieve a better ARL performance. Sim (2003) studied the performance of the synthetic chart based on the gamma and exponential distributions for known and unknown parameters, respectively and concluded that the synthetic chart outperforms the Shewhart $\bar{X}$ chart with either asymmetric probability limits or 3-sigma control limits. Scariano & Calzada (2003) discussed a synthetic chart for exponential data, derived an expression for its ARL and design parameters and showed that the chart outperforms the Shewhart chart for individuals but is inferior to the EWMA and cumulative sum (CUSUM) charts in detecting decreases in the exponential mean. Huang & Chen (2005) suggested a synthetic chart for monitoring process dispersion by combining the sample standard deviation, S chart and the CRL chart. Chen & Huang (2005) combined the sample range, R chart and the CRL chart to form a synthetic chart for process dispersion. Costa & Rahim (2006) proposed a synthetic chart based on a noncentral chi-square statistic that is superior to the joint $\bar{X}$ and R chart in detecting shifts in the mean and/or standard deviation. Costa, Magalhaes & Epprecht (2006) considered a synthetic chart with two-stage sampling to monitor the process mean and variance, and claimed that the chart is more convenient to administer than the joint $\bar{X}$ and S chart with double sampling, although both charts have similar performances.

Similar to the $\bar{X}$, EWMA and CUSUM charts, the synthetic chart for the mean proposed by Wu & Spedding (2000*b*) requires the assumption that the distribution of the quality characteristic is normal or approximately normal. But in some situations, it may happen that this condition does not hold (for instance, see Jacobs (1990)). Experience in the chemical industry shows that there are a number of reasons why a process that is operating in a state of statistical process control, yields non normal skewed distributions. Some of these reasons are:

- measurements or operation in the vicinity of a material's physical limits, e.g. saturation, phase change, boiling point, tensile strength.
- measurements of a characteristic that has zero as a natural limit, e.g. moisture content, impurity content, warpage, bow.
- mathematical relationships between variables, e.g. a variable with an Arrhenius-type exponential dependence on process temperature.

To deal with nonnormal underlying distributions, the approaches that are currently used are (i) transforming the data to attain an approximate normal distribution, (ii) increasing the sample size so that the sample average follows an approximate normal distribution, and (iii) employing heuristic control charts for skewed populations. The existing heuristic charts for skewed populations are the $\bar{X}$ and R charts based on the weighted variance (WV) method proposed by Bai & Choi (1995), the $\bar{X}$ chart based on the scaled weighted variance (SWV) method suggested by Castagliola (2000), the $\bar{X}$, CUSUM and EWMA charts using the weighted standard deviation method presented by Chang & Bai (2001) and the $\bar{X}$ and R charts based on the skewness correction method proposed by Chan & Cui (2003).

Some of the other works on control charts for skewed populations are made by (i) Schneider, Kasperski, Ledford & Kraushaar (1995) who discussed methods to establish control limits when the data are positively skewed and censored from below, (ii) Wu (1996) who proposed an approach to optimize the control limits of the $\bar{X}$ chart for skewed populations so that the average number of scrap products is minimized without increasing the Type-I error, (iii) Dou & Sa (2002) who suggested a procedure to construct a one-sided $\bar{X}$ chart for positively skewed distributions using the Edgeworth expansion method, (iv) Chen (2004) who presented an economic design of $\bar{X}$ charts for nonnormal data using variable sampling policy, (v) Nichols & Padgett (2005) who considered a bootstrap control chart for Weibull percentiles, (vi) Kan & Yazici (2006) who proposed a skewness correction method in setting the asymmetric limits of the individuals charts for Burr and Weibull distributed data, and (vii) Tsai (2007) who developed two control charts and process capability ratios based on the skew normal distribution to monitor the process mean and evaluate the process capability of nonnormal data.

Recently, Khoo, Z.Wu & Atta (2008) proposed a synthetic control chart for monitoring shifts in the process mean of skewed populations using the WV method, where no assumption of the distribution of the underlying process is needed. This chart was shown to provide vast improvements over all the existing charts for skewed populations, in terms of false alarm and mean shift detection rates for cases with known and unknown parameters.

This paper extends the work of Khoo et al. (2008) by proposing a synthetic Scaled WV (SWV) control chart for monitoring the mean of skewed populations. The synthetic SWV-$\bar{X}$ chart will be shown to outperform the synthetic WV-$\bar{X}$ chart of Khoo et al. (2008) for the case with a negative shift in the mean, when the same in-control ARL is considered for the two charts. For this case, the superiority of the synthetic SWV-$\bar{X}$ chart increases with the level of skewness. Note that for a positive shift in the mean, the synthetic SWV-$\bar{X}$ chart is only slightly less effective than the synthetic WV-$\bar{X}$ chart. Thus, for a process having a skewed population, where past experience indicates that whenever a signal is triggered a negative shift usually occurs, then the synthetic SWV-$\bar{X}$ chart can be a favourable substitute for the synthetic WV-$\bar{X}$ chart. The rest of this paper is organized as follows : Section 2 gives a review on the synthetic $\bar{X}$, the WV-$\bar{X}$ and the SWV-$\bar{X}$ charts. Section 3 presents the proposed synthetic SWV-$\bar{X}$ chart and details the methodology used for comparing both the synthetic WV-$\bar{X}$ and synthetic SWV-$\bar{X}$ charts. Section 4 illustrates the use of the synthetic SWV-$\bar{X}$ with an example. Section 5 completes the paper with the main conclusions drawn from our study.

2 Literature review

2.1 Synthetic $\bar{X}$ control chart

Let us consider firstly that the quality characteristic X is a normal, $N(\mu, \sigma)$ random variable, where μ is the in-control mean and σ is the in-control standard-deviation. The synthetic $\bar{X}$ chart, introduced by Wu & Spedding (2000b), makes a Shewhart $\bar{X}$ chart and a conforming run length (CRL) chart work together. The synthetic $\bar{X}$ chart comprises a $\bar{X}/S$ sub-chart and a CRL/S sub-chart. The CRL is defined as the number of inspected units between two consecutive nonconforming units (including the ending nonconforming unit). Figure 1 is an example that shows how the CRL value is determined, assuming that a process starts at $t = 0$. Here, $CRL_1 = 5$, $CRL_2 = 2$ and $CRL_3 = 4$. The operation of the synthetic $\bar{X}$ chart is based on the following steps:

Step 1 : Set the lower control limit: $L \in \{1, 2, \ldots\}$ of the CRL/S sub-chart and set the constant $K > 0$ of the $\bar{X}/S$ sub-chart defined by the following control limits:

$$LCL_{\bar{X}} = \mu - K\sigma \tag{1}$$

$$UCL_{\bar{X}} = \mu + K\sigma \tag{2}$$

Step 2 : Take a random sample of n observations at each inspection point and compute the sample mean, $\bar{X}$.

Step 3 : If $LCL_{\bar{X}} < \bar{X} < UCL_{\bar{X}}$, the sample is considered as a conforming sample and the control flow moves back to Step 2. Otherwise, the sample is a nonconforming sample and the control flow advances to Step 4.

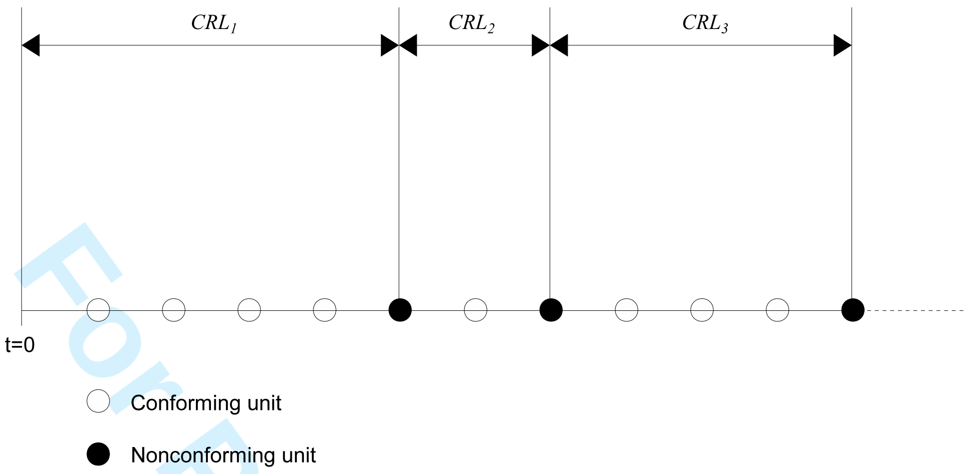

Figure 1: Conforming Run Length

Step 4 : Count the number of $\bar{X}$ samples between the current and the last nonconforming sample (which includes the current but excludes the last nonconforming sample) as the CRL value of the CRL/S sub-chart.

Step 5 : If the value of $CRL \geq L$, the process is declared in-control and the control flow moves back to Step 2. Otherwise, the process is out-of-control and the control flow advances to Step 6.

Step 6 : Signals an out-of-control status to indicate a process mean shift.

Step 7 : Find and remove assignable cause(s). Then move back to Step 2.

Wu & Spedding (2000b) demonstrated that the Average Run Length (ARL) of the synthetic $\bar{X}$ control chart corresponding to specific values of K, L, n and $\delta = |\mu - \mu_1|/\sigma$ (desired magnitude of standardized mean shift) is equal to

$$ARL(\delta) = \frac{1}{\pi(1-(1-\pi)^L)} \tag{3}$$

with

$$\pi = \Phi(-(K+\delta)\sqrt{n}) + \Phi(-(K-\delta)\sqrt{n}) \tag{4}$$

where $\Phi(.)$ is the standard normal distribution function. In particular, for $\delta = 0$, we have

$$ARL(0) = \frac{1}{2\Phi(-K\sqrt{n})(1-(1-2\Phi(-K\sqrt{n}))^L)}$$

Using these equations, Wu & Spedding (2000b) suggested optimal combinations of K and L (useful for the Step 1 described above) that minimize the out-of-control ARL for desired magnitudes of the standardized mean shift δ and an in-control ARL (ARL_0) of interest.

2.2 The Weighted Variance and Scaled Weighted Variance $\bar{X}$ charts

Now, let us consider that the distribution of the quality characteristic X is no longer normal but is some continuous unimodal skew distribution $f_X(x)$, where $\mu = E(X)$ is the in-control mean, $\sigma = \sigma(X)$ is the the in-control standard-deviation and $\theta = P(X \leq \mu)$ is the in-control probability that X is less than or equal to the mean μ. The Weighted Variance $\bar{X}$ chart (WV-$\bar{X}$ chart in short) was initially proposed by Choobineh & Ballard (1987) who suggested the use of the semivariance approximation of Choobineh & Branting (1986) in order to provide control limits for the mean in the case of a quality characteristic having a skew distribution. Bai & Choi (1995) provided computations and tables to simplify the implementation of the WV-$\bar{X}$ chart proposed by Choobineh & Ballard (1987). The control limits of the WV-$\bar{X}$ chart are

$$LCL^{WV} = \mu - K_L^{WV}\sigma \quad (5)$$
$$UCL^{WV} = \mu + K_U^{WV}\sigma \quad (6)$$

where K_L^{WV} and K_U^{WV} are equal to

$$K_L^{WV} = \Phi^{-1}(1-\tfrac{\alpha}{2})\sqrt{\frac{2(1-\theta)}{n}} \quad (7)$$
$$K_U^{WV} = \Phi^{-1}(1-\tfrac{\alpha}{2})\sqrt{\frac{2\theta}{n}} \quad (8)$$

where n is the sample size, $\Phi^{-1}(.)$ the inverse standard normal distribution function and α is a desired Type-I error. It is worth to note that if $\theta = \frac{1}{2}$

(distribution of X is symmetrical) then

$$K_L^{WV} = K_U^{WV} = \frac{\Phi^{-1}(1-\frac{\alpha}{2})}{\sqrt{n}}$$

and the control limits in equation (5) and equation (6) are reduced to the classical Shewhart $\bar{X}$ control limits. The Scaled Weighted Variance $\bar{X}$ chart (SWV-$\bar{X}$ chart in short) was suggested by Castagliola (2000) as an improvement over the WV-$\bar{X}$ chart. Castagliola (2000) provided explanations concerning the shortcomings of the WV method and how these shortcomings were addressed using the SWV method. The control limits of the SWV-$\bar{X}$ chart are as follow (see Castagliola (2000)):

$$\begin{aligned} LCL^{SWV} &= \mu - K_L^{SWV}\sigma && (9) \\ UCL^{SWV} &= \mu + K_U^{SWV}\sigma && (10) \end{aligned}$$

where K_L^{SWV} and K_U^{SWV} are equal to

$$\begin{aligned} K_L^{SWV} &= \Phi^{-1}\left(1-\frac{\alpha}{4\theta}\right)\sqrt{\frac{1-\theta}{n\theta}} && (11) \\ K_U^{SWV} &= \Phi^{-1}\left(1-\frac{\alpha}{4(1-\theta)}\right)\sqrt{\frac{\theta}{n(1-\theta)}} && (12) \end{aligned}$$

It is worth to note that the two constants above can only be computed if $\frac{\alpha}{4} < \theta < 1 - \frac{\alpha}{4}$ and, as for the WV-$\bar{X}$ chart, if $\theta = \frac{1}{2}$ then

$$K_L^{SWV} = K_U^{SWV} = \frac{\Phi^{-1}(1-\frac{\alpha}{2})}{\sqrt{n}}$$

and the control limits in equation (9) and equation (10) are also reduced to the classical Shewhart $\bar{X}$ control limits.

3 The Synthetic Weighted Variance and Synthetic Scaled Weighted Variance $\bar{X}$ charts

The synthetic WV-$\bar{X}$ chart, suggested by Khoo et al. (2008), is based on the idea of integrating the WV method of Bai & Choi (1995) with the synthetic $\bar{X}$ chart of Wu & Spedding (2000*b*). The operation of the synthetic WV-$\bar{X}$ chart is similar to that of the synthetic $\bar{X}$ chart described in section 2.1, except that the control limits in equation (1) and equation (2) are replaced with the control limits in equation (5) and equation (6) for the WV-$\bar{X}/S$ sub chart. Khoo et al. (2008) compared by simulation the synthetic WV-$\bar{X}$ chart with different other alternatives (i.e. SC-$\bar{X}$ chart by Chan & Cui (2003) and the WSD-$\bar{X}$, WSD-CUSUM and WSD-EWMA charts by Chang & Bai (2001)) and concluded that the former gives the most favourable results, in terms of false alarms and mean shift detection rates, in both the known and unknown parameter cases, where the results of the synthetic WV-$\bar{X}$ chart are even better when the skewness of the underlying distribution is larger.

In this paper, we suggest to integrate the SWV method of Castagliola (2000) with the synthetic $\bar{X}$ chart of Wu & Spedding (2000b) by replacing the control limits in equation (1) and equation (2) with the control limits in equation (9) and equation (10) for the SWV-$\bar{X}/S$ sub chart. The resulting chart will be called a synthetic SWV-$\bar{X}$ chart. The goal of this paper is to evaluate the respective efficiencies of both the synthetic WV-$\bar{X}$ and synthetic SWV-$\bar{X}$ charts in terms of the out-of-control ARL. In order to compare these two charts, we have chosen an innovative methodology that does not involve any simulation. This methodology is decribed below:

1. For the sake of simplicity, we assume that $\mu = 0$ and $\sigma = 1$.

2. Let $\beta = E((\frac{X-\mu}{\sigma})^3)$ and $\psi = E((\frac{X-\mu}{\sigma})^4) - 3$ be the skewness and kurtosis coefficients, respectively of the quality characteristic X. In our study, we restrict the values of the skewness coefficient $\beta \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5\}$ and, for each of these values, we select 7 different values $\psi_1, \ldots, \psi_7$ for the kurtosis coefficient ψ. As Figure 2 clearly shows, for each selected skewness coefficient β, the 6 smaller kurtosis coefficients $\psi_1, \ldots, \psi_6$ are uniformly distributed within the curve corresponding to the lower limit for any possible distributions and the curve corresponding to the lognormal distribution while the largest kurtosis coefficient ψ_7 is just above the curve corresponding to the lognormal distribution. This strategy guarantees to cover a large spectrum of distributions, including the gamma, Weibull

and lognormal distributions. The values of the skewness β and kurtosis ψ coefficients used in this paper are listed in Table 1.

3. For each combination (β, ψ_i), for $i = 1, \ldots, 7$, in Table 1, we compute the parameters (a_i, b_i, c_i, d_i) of the Johnson (1949) distribution having $\mu = 0$ for mean, $\sigma = 1$ for standard-deviation, β for skewness coefficient and ψ_i, $i = 1, \ldots, 7$ for kurtosis coefficient (the estimation algorithm is due to Hill, Hill & Holder (1976)). Based on Johnson's work, we know that there is an unique set of parameters (a_i, b_i, c_i, d_i) satisfying this condition (the main properties of the Johnson system of distributions are summarized in the Appendix). Let $F_J(x|a, b, c, d)$ be the Johnson distribution function of parameters (a, b, c, d). For each $\beta \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5\}$, we can compute $\theta_i = P(X \leq \mu) = F_J(\mu|a_i, b_i, c_i, d_i)$ and $\bar{\theta} = \frac{1}{7}(\theta_1 + \cdots + \theta_7)$ being the average probability that X is less than or equal to the mean μ over the spectrum of considered distributions.

4. If the distribution of the quality characteristic X is known, the distribution of the sample mean $\bar{X}$ is generally unknown (except for some rare cases) and, therefore, there is no closed-form for it. Nevertheless, it is well known that the mean $\mu_{\bar{X}}$, the standard-deviation $\sigma_{\bar{X}}$, the skewness coefficient $\beta_{\bar{X}}$ and the kurtosis coefficient $\psi_{\bar{X}}$ of $\bar{X}$ are related to μ, σ, β and ψ through the following simple formulae:

$$\begin{aligned} \mu_{\bar{X}} &= \mu \\ \sigma_{\bar{X}} &= \frac{\sigma}{\sqrt{n}} \\ \beta_{\bar{X}} &= \frac{\beta}{\sqrt{n}} \\ \psi_{\bar{X}} &= \frac{\psi}{n} \end{aligned}$$

Consequently, if the distribution of the sample mean $\bar{X}$ is actually unknown, we simply suggest to approximate it with the unique Johnson distribution with parameters $(a_{\bar{X}}, b_{\bar{X}}, c_{\bar{X}}, d_{\bar{X}})$ having $\mu_{\bar{X}}$ for mean, $\sigma_{\bar{X}}$ for standard-deviation, $\beta_{\bar{X}}$ for skewness coefficient and $\psi_{\bar{X}}$ for kurtosis coefficient and estimated with the algorithm of Hill et al. (1976). In order

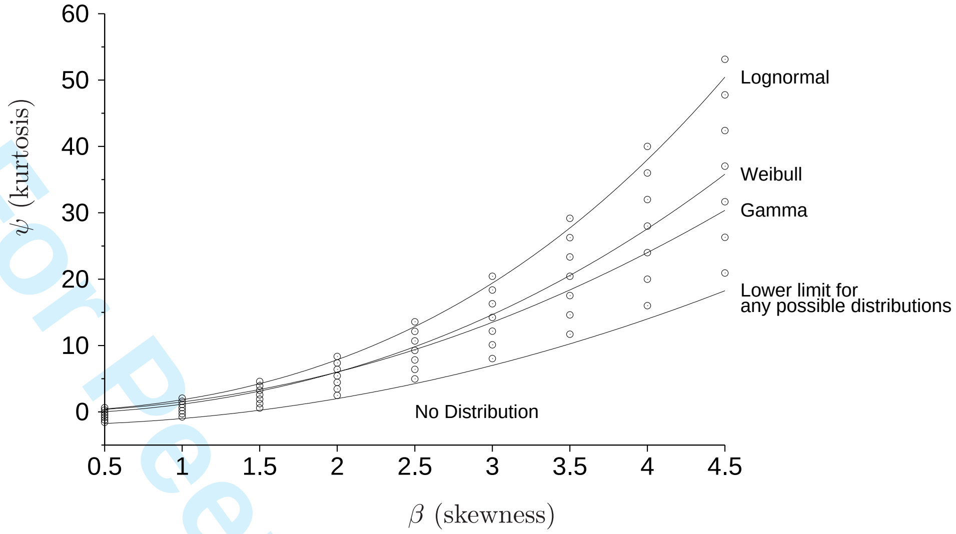

Figure 2: Selected skewness β and kurtosis ψ_i coefficients covering the area corresponding to the gamma, the Weibull and the lognormal distributions.

	ψ						
β	ψ_1	ψ_2	ψ_3	ψ_4	ψ_5	ψ_6	ψ_7
0.5	-1.5669	-1.2006	-0.8343	-0.4680	-0.1017	0.2646	0.6309
1.0	-0.7642	-0.2927	0.1789	0.6504	1.1220	1.5935	2.0651
1.5	0.5834	1.2501	1.9168	2.5835	3.2502	3.9170	4.5837
2.0	2.4886	3.4659	4.4431	5.4204	6.3976	7.3748	8.3521
2.5	4.9666	6.3997	7.8328	9.2659	10.6991	12.1322	13.5653
3.0	8.0333	10.1000	12.1666	14.2333	16.3000	18.3666	20.4333
3.5	11.7056	14.6167	17.5278	20.4389	23.3500	26.2612	29.1723
4.0	16.0000	20.0000	24.0000	28.0000	32.0000	36.0000	40.0000
4.5	20.9333	26.2999	31.6665	37.0332	42.3998	47.7664	53.1330

Table 1: Skewness β and kurtosis ψ_i coefficients used for the comparison of the synthetic WV-$\bar{X}$ and synthetic SWV-$\bar{X}$ charts

to validate this approach, we conducted a thorough study (not presented here) where we computed for different combinations of (β, ψ), the cumulative distribution function of $\bar{X}$ either by intensive simulations or by fitting with a Johnson distribution. The result of this study clearly demonstated that the cumulative distribution function of $\bar{X}$ obtained by fitting a Johnson distribution is extremely close to the real one obtained by simulation, thus providing an easy-to-use and accurate approximation for the cumulative distribution function of $\bar{X}$.

5. For a combination (β, ψ_i), the ARL of both the Synthetic WV-$\bar{X}$ and Synthetic SWV-$\bar{X}$ charts are computed using equation (3) where π in equation (4) is replaced by

$$\pi = F_J(K_L|a_{\bar{X}}, b_{\bar{X}}, c_{\bar{X}} + \delta, d_{\bar{X}}) + 1 - F_J(K_U|a_{\bar{X}}, b_{\bar{X}}, c_{\bar{X}} + \delta, d_{\bar{X}}).$$

Here (K_L, K_U) are the constants (K_L^{WV}, K_U^{WV}) for the Synthetic WV-$\bar{X}$ chart or (K_L^{SWV}, K_U^{SWV}) for the Synthetic SWV-$\bar{X}$ chart. Consquently, for values of L, K_L, K_U and β, we can compute 7 differents values $ARL_1, \ldots, ARL_7$ corresponding to the 7 kurtosis $\psi_1, \ldots, \psi_7$ and we can also compute $\overline{ARL} = \frac{1}{7}(ARL_1 + \cdots + ARL_7)$ as the average ARL over the spectrum of considered distributions.

In Tables 2, 3 and 4 we have computed the values of the constants K_L, K_U and L, for both the Synthetic WV-$\bar{X}$ chart and the Synthetic SWV-$\bar{X}$ chart, for $n = 5$, $\beta \in \{0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5\}$ and for $\delta \in \{-1.5,\ -1,\ -0.7,\ -0.5,$ $-0.4, -0.3, -0.2, -0.1, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1, 1.5\}$. The average in-control ARL is $\overline{ARL}_0 = 370.4$. For example, if the value of the skewness coefficient is $\beta = 1.5$ then the average probability that X is less than or equal to the mean μ is $\bar{\theta} = 0.636$. If we want to detect a standardized mean shift $\delta = -0.5$ (i.e. a decrease of 0.5σ), we have $K_L = 0.851$, $K_U = 1.126$, $L = 7$ for the Synthetic WV-$\bar{X}$ chart and the average out-of-control ARL is $\overline{ARL} = 5.1$ while, for the Synthetic SWV-$\bar{X}$ chart, we have $K_L = 0.789$, $K_U = 1.252$, $L = 9$ and the average out-of-control ARL is $\overline{ARL} = 3.7$.

In Tables 2, 3 and 4, the $\overline{ARL}$ values in bold characters correspond to the lowest out-of-control average ARL's. This clearly demonstrates that when the standardized mean shift $\delta < 0$, the Synthetic SWV-$\bar{X}$ chart always have smaller average out-of-control ARL than the Synthetic WV-$\bar{X}$ chart. When the standardized mean shift $\delta > 0$, the previous conclusion is reversed.

4 An illustrative example

In order to illustrate the use of the Synthetic SWV-$\bar{X}$ chart, let us consider a 125g yogurt cup filling process where the quality characteristic X is the weight of each yogurt cup. A long term study (Phase I, realized in a local company) based on a large database of yogurt cup weights showed that the distribution of the quality characteristic X is significantly skewed. This study also allowed accurate estimations of the in-control mean $\mu = 124.9$, the in-control standard-deviation $\sigma = 0.76$ and the in-control probability $\theta = 0.679$ that X is less than or equal to its mean μ. The quality practitioner in charge of this process decided to take $n = 5$ yogurt cups every hour. Based on Table 3, he decided to choose the value of $\theta = 0.682$ (which is the closest to the in-control probability $\theta = 0.679$) and to use the constants K_L, K_U and L optimally designed for detecting a mean shift $\delta = -0.3$, i.e. $K_L = 0.701$, $K_U = 1.306$ and $L = 9$, yielding the following Synthetic SWV-$\bar{X}$ control limits

$$\begin{aligned} LCL &= 124.9 - 0.701 \times 0.76 = 124.37 \\ UCL &= 124.9 + 1.306 \times 0.76 = 125.89 \end{aligned}$$

In Table 5, we recorded 30 samples corresponding to a 30 hours sequence of production (Phase II) from the 101th hour to the 130th hour. In each row we have the values corresponding to $n = 5$ yogurt cups weighed every hour. The last column is the mean $\bar{X}_i$ of these $n = 5$ values. The samples in Table 5 are also plotted in Figure 3 (top). In Figure 3 (bottom), we plotted the mean $\bar{X}_i$ of the 30 samples with the control limits $LCL = 124.37$ and $UCL = 125.89$ of the SWV-$\bar{X}/S$ sub chart. Concerning the values of $\bar{X}_i$, for $i = 1, \ldots, 100$, (corresponding to the starting phase of the process, but not recorded in Table 5), they all verify that $LCL < \bar{X}_i < UCL$. As we can see in Figure 3 (bottom),

			WV-$\bar{X}$ chart				SWV-$\bar{X}$ chart			
β	$\bar{\theta}$	δ	K_L	K_U	L	$\overline{ARL}$	K_L	K_U	L	$\overline{ARL}$
0.5	0.554	-1.5	0.868	0.968	2	1.1	0.843	1.004	2	**1.1**
		-1.0	0.918	1.023	4	1.8	0.890	1.063	4	**1.7**
		-0.7	0.955	1.065	7	3.7	0.925	1.108	7	**3.3**
		-0.5	0.994	1.109	13	8.7	0.960	1.153	13	**7.3**
		-0.4	1.020	1.137	20	16.0	0.984	1.183	20	**12.6**
		-0.3	1.043	1.163	30	34.3	1.009	1.215	33	**25.0**
		-0.2	1.056	1.177	38	89.1	1.037	1.251	60	**58.8**
		-0.1	0.972	1.083	9	236.8	1.057	1.277	98	**164.8**
		0.1	1.145	1.277	263	**191.5**	1.110	1.344	472	261.6
		0.2	1.114	1.242	125	**90.9**	1.082	1.309	190	127.7
		0.3	1.083	1.208	65	**45.6**	1.054	1.273	90	62.5
		0.4	1.054	1.175	37	**24.7**	1.026	1.237	47	32.6
		0.5	1.028	1.146	23	**14.3**	0.999	1.203	27	18.3
		0.7	0.984	1.097	11	**5.9**	0.956	1.148	12	7.0
		1.0	0.933	1.040	5	**2.3**	0.904	1.081	5	2.6
		1.5	0.868	0.968	2	**1.1**	0.843	1.004	2	1.2
1.0	0.600	-1.5	0.817	1.001	2	1.1	0.778	1.077	2	**1.1**
		-1.0	0.863	1.058	4	1.6	0.818	1.140	4	**1.5**
		-0.7	0.889	1.089	6	2.9	0.839	1.174	6	**2.6**
		-0.5	0.920	1.128	10	6.5	0.869	1.221	11	**5.0**
		-0.4	0.936	1.147	13	12.0	0.886	1.248	16	**8.1**
		-0.3	0.944	1.158	15	27.2	0.905	1.278	25	**15.5**
		-0.2	0.914	1.120	9	77.9	0.927	1.313	44	**37.0**
		-0.1	0.817	1.001	2	210.6	0.935	1.327	55	**121.9**
		0.1	1.081	1.326	177	**176.6**	1.003	1.432	514	244.9
		0.2	1.052	1.290	100	**88.5**	0.984	1.403	248	132.8
		0.3	1.024	1.256	60	**47.4**	0.965	1.373	130	72.4
		0.4	0.998	1.224	38	**27.1**	0.946	1.343	73	41.1
		0.5	0.974	1.195	25	**16.4**	0.926	1.312	43	24.4
		0.7	0.931	1.141	12	**7.0**	0.888	1.252	17	9.8
		1.0	0.877	1.075	5	**2.7**	0.839	1.174	6	3.4
		1.5	0.817	1.001	2	**1.2**	0.778	1.077	2	1.3
1.5	0.636	-1.5	0.770	1.019	2	1.1	0.723	1.127	2	**1.1**
		-1.0	0.796	1.053	3	1.5	0.755	1.188	4	**1.4**
		-0.7	0.829	1.097	5	2.5	0.772	1.221	6	**2.1**
		-0.5	0.851	1.126	7	5.1	0.789	1.252	9	**3.7**
		-0.4	0.860	1.138	8	9.4	0.803	1.279	13	**5.7**
		-0.3	0.841	1.113	6	21.9	0.817	1.306	19	**10.3**
		-0.2	0.796	1.053	3	64.2	0.833	1.335	29	**24.5**
		-0.1	0.724	0.959	1	183.3	0.819	1.309	20	**96.7**
		0.1	1.046	1.384	129	**180.6**	0.913	1.485	328	218.1
		0.2	1.012	1.339	75	**94.8**	0.897	1.456	193	124.3
		0.3	0.981	1.298	47	**52.8**	0.881	1.425	116	72.1
		0.4	0.953	1.261	31	**30.9**	0.865	1.396	74	43.4
		0.5	0.929	1.230	22	**18.9**	0.850	1.368	48	27.1
		0.7	0.882	1.166	11	**8.1**	0.823	1.316	22	11.6
		1.0	0.829	1.097	5	**3.0**	0.784	1.243	8	4.1
		1.5	0.770	1.019	2	**1.2**	0.723	1.127	2	1.3

Table 2: Constants K_L, K_U and L, for both the Synthetic WV-$\bar{X}$ and Synthetic SWV-$\bar{X}$ charts, for $n = 5$, $\beta \in \{0.5, 1, 1.5\}$ and $\overline{ARL}_0 = 370.4$.

			WV-$\bar{X}$ chart				SWV-$\bar{X}$ chart			
β	$\bar{\theta}$	δ	K_L	K_U	L	$\overline{ARL}$	K_L	K_U	L	$\overline{ARL}$
2.0	0.663	-1.5	0.730	1.024	2	1.1	0.672	1.141	2	**1.1**
		-1.0	0.759	1.065	3	1.4	0.689	1.177	3	**1.3**
		-0.7	0.781	1.095	4	2.2	0.709	1.222	5	**1.8**
		-0.5	0.798	1.119	5	4.4	0.723	1.251	7	**3.0**
		-0.4	0.798	1.119	5	8.0	0.733	1.274	9	**4.4**
		-0.3	0.759	1.065	3	18.6	0.745	1.300	12	**7.7**
		-0.2	0.730	1.024	2	53.4	0.751	1.314	14	**19.4**
		-0.1	0.684	0.959	1	159.4	0.709	1.222	5	**86.7**
		0.1	1.040	1.459	114	**190.6**	0.866	1.558	281	213.5
		0.2	0.998	1.400	63	**105.7**	0.850	1.524	170	126.6
		0.3	0.961	1.348	39	**61.2**	0.831	1.485	102	76.8
		0.4	0.929	1.303	26	**36.8**	0.814	1.446	64	47.7
		0.5	0.899	1.261	18	**22.8**	0.797	1.411	42	30.4
		0.7	0.852	1.195	10	**9.6**	0.766	1.345	20	13.3
		1.0	0.798	1.119	5	**3.4**	0.728	1.263	8	4.6
		1.5	0.730	1.024	2	**1.2**	0.672	1.141	2	1.4
2.5	0.682	-1.5	0.705	1.033	2	1.1	0.632	1.139	2	**1.1**
		-1.0	0.738	1.081	3	1.3	0.651	1.185	3	**1.2**
		-0.7	0.762	1.117	4	2.0	0.664	1.216	4	**1.7**
		-0.5	0.762	1.117	4	3.9	0.682	1.261	6	**2.6**
		-0.4	0.738	1.081	3	7.1	0.689	1.278	7	**3.8**
		-0.3	0.705	1.033	2	16.4	0.701	1.306	9	**6.7**
		-0.2	0.652	0.956	1	45.3	0.689	1.278	7	**18.3**
		-0.1	0.652	0.956	1	164.1	0.632	1.139	2	**85.2**
		0.1	1.071	1.570	144	**202.5**	0.845	1.647	286	215.2
		0.2	1.010	1.481	64	**119.3**	0.829	1.609	174	133.7
		0.3	0.953	1.398	33	**72.1**	0.810	1.563	104	85.1
		0.4	0.913	1.338	21	**44.4**	0.789	1.515	63	55.0
		0.5	0.875	1.283	14	**27.9**	0.769	1.468	40	35.9
		0.7	0.823	1.207	8	**11.7**	0.733	1.383	18	16.0
		1.0	0.762	1.117	4	**3.8**	0.682	1.261	6	5.3
		1.5	0.652	0.956	1	**1.1**	0.632	1.139	2	1.3
3.0	0.697	-1.5	0.687	1.042	2	1.1	0.607	1.146	2	**1.1**
		-1.0	0.724	1.098	3	1.3	0.627	1.199	3	**1.2**
		-0.7	0.724	1.098	3	1.9	0.641	1.237	4	**1.6**
		-0.5	0.724	1.098	3	3.6	0.652	1.266	5	**2.4**
		-0.4	0.687	1.042	2	6.6	0.661	1.290	6	**3.4**
		-0.3	0.629	0.955	1	15.0	0.669	1.310	7	**6.2**
		-0.2	0.629	0.955	1	42.0	0.627	1.199	3	**18.4**
		-0.1	0.629	0.955	1	229.9	0.574	1.058	1	**87.9**
		0.1	1.123	1.703	225	**213.6**	0.841	1.749	326	220.3
		0.2	1.052	1.595	85	**134.1**	0.826	1.709	200	144.3
		0.3	0.956	1.450	30	**84.7**	0.804	1.654	114	96.4
		0.4	0.893	1.354	16	**53.6**	0.779	1.590	64	64.6
		0.5	0.854	1.296	11	**34.1**	0.755	1.531	39	43.2
		0.7	0.792	1.201	6	**14.2**	0.711	1.417	16	19.5
		1.0	0.724	1.098	3	**4.3**	0.661	1.290	6	6.2
		1.5	0.629	0.955	1	**1.1**	0.574	1.058	1	1.3

Table 3: Constants K_L, K_U and L, for both the Synthetic WV-$\bar{X}$ and Synthetic SWV-$\bar{X}$ charts, for $n = 5$, $\beta \in \{2, 2.5, 3\}$ and $\overline{ARL}_0 = 370.4$.

			WV-$\bar{X}$ chart				SWV-$\bar{X}$ chart			
β	$\bar{\theta}$	δ	K_L	K_U	L	$\overline{ARL}$	K_L	K_U	L	$\overline{ARL}$
3.5	0.708	-1.5	0.674	1.049	2	1.1	0.589	1.153	2	**1.0**
		-1.0	0.714	1.113	3	1.3	0.610	1.212	3	**1.2**
		-0.7	0.714	1.113	3	1.8	0.625	1.254	4	**1.5**
		-0.5	0.674	1.049	2	3.4	0.636	1.287	5	**2.2**
		-0.4	0.674	1.049	2	6.0	0.636	1.287	5	**3.2**
		-0.3	0.611	0.951	1	13.2	0.636	1.287	5	**5.9**
		-0.2	0.611	0.951	1	43.3	0.589	1.153	2	**18.6**
		-0.1	0.611	0.951	1	239.9	0.553	1.050	1	**145.8**
		0.1	1.174	1.829	328	**223.0**	0.848	1.858	405	226.5
		0.2	1.117	1.740	139	**148.6**	0.834	1.823	258	155.9
		0.3	0.931	1.450	21	**97.6**	0.812	1.763	141	109.5
		0.4	0.859	1.337	11	**62.9**	0.775	1.663	63	76.0
		0.5	0.807	1.257	7	**40.4**	0.739	1.566	32	51.6
		0.7	0.745	1.161	4	**16.8**	0.688	1.430	13	23.4
		1.0	0.674	1.049	2	**4.8**	0.625	1.254	4	7.2
		1.5	0.611	0.951	1	**1.1**	0.553	1.050	1	1.2
4.0	0.717	-1.5	0.665	1.057	2	1.1	0.574	1.155	2	**1.0**
		-1.0	0.665	1.057	2	1.2	0.596	1.222	3	**1.2**
		-0.7	0.709	1.128	3	1.7	0.596	1.222	3	**1.5**
		-0.5	0.665	1.057	2	3.2	0.612	1.270	4	**2.1**
		-0.4	0.665	1.057	2	5.8	0.625	1.307	5	**3.0**
		-0.3	0.596	0.948	1	12.1	0.612	1.270	4	**5.7**
		-0.2	0.596	0.948	1	66.6	0.538	1.047	1	**19.2**
		-0.1	0.596	0.948	1	267.0	0.538	1.047	1	**181.8**
		0.1	1.220	1.941	412	**232.8**	0.859	1.967	493	234.0
		0.2	1.180	1.878	214	**163.1**	0.849	1.939	335	168.7
		0.3	0.893	1.421	14	**110.2**	0.832	1.892	201	124.4
		0.4	0.825	1.312	8	**72.2**	0.789	1.773	78	90.3
		0.5	0.769	1.223	5	**46.9**	0.723	1.588	25	62.6
		0.7	0.709	1.128	3	**19.5**	0.653	1.388	8	28.0
		1.0	0.665	1.057	2	**5.3**	0.574	1.155	2	8.1
		1.5	0.596	0.948	1	**1.0**	0.538	1.047	1	1.2
4.5	0.723	-1.5	0.659	1.064	2	1.1	0.562	1.155	2	**1.0**
		-1.0	0.659	1.064	2	1.2	0.586	1.229	3	**1.2**
		-0.7	0.659	1.064	2	1.7	0.586	1.229	3	**1.4**
		-0.5	0.659	1.064	2	3.1	0.604	1.282	4	**2.0**
		-0.4	0.584	0.943	1	5.4	0.604	1.282	4	**2.9**
		-0.3	0.584	0.943	1	11.5	0.586	1.229	3	**5.7**
		-0.2	0.584	0.943	1	133.1	0.524	1.035	1	**18.9**
		-0.1	0.584	0.943	1	272.6	0.524	1.035	1	**173.4**
		0.1	1.265	2.044	453	**244.2**	0.876	2.076	568	243.7
		0.2	0.959	1.550	22	**179.0**	0.868	2.053	409	182.8
		0.3	0.857	1.384	10	**121.4**	0.854	2.014	265	140.1
		0.4	0.791	1.279	6	**80.7**	0.813	1.897	105	105.9
		0.5	0.741	1.197	4	**52.9**	0.676	1.499	12	74.1
		0.7	0.659	1.064	2	**21.9**	0.604	1.282	4	32.2
		1.0	0.584	0.943	1	**5.5**	0.524	1.035	1	8.5
		1.5	0.584	0.943	1	**1.0**	0.524	1.035	1	1.1

Table 4: Constants K_L, K_U and L, for both the Synthetic WV-$\bar{X}$ and Synthetic SWV-$\bar{X}$ charts, for $n = 5$, $\beta \in \{3.5, 4, 4.5\}$ and $\overline{ARL}_0 = 370.4$.

i	X					$\bar{X}_i$	CRL
101	125.3	124.9	124.4	124.6	125.2	124.88	···
102	124.8	124.8	126.4	124.6	126.4	125.40	···
103	124.7	124.4	124.7	125.2	128.6	125.52	···
104	125.5	124.6	124.8	124.4	124.3	124.72	···
105	124.7	124.3	124.6	125.0	124.4	124.60	···
106	124.5	124.8	124.7	124.4	124.9	124.66	···
106	124.4	125.1	125.6	124.5	124.8	124.88	···
107	124.4	125.3	124.3	125.0	124.3	124.66	···
108	124.5	126.8	124.4	125.9	124.8	125.28	···
110	124.4	125.4	124.4	125.0	129.2	125.68	···
111	126.2	124.3	125.4	124.9	124.5	125.06	···
112	124.4	124.2	124.3	124.4	124.3	**124.32**	**112**
113	124.6	124.6	124.8	124.4	124.5	124.58	···
114	124.4	124.5	124.7	124.6	126.1	124.86	···
115	125.3	124.8	124.4	124.7	124.5	124.74	···
116	124.2	124.9	125.4	124.1	124.6	124.64	···
117	125.0	124.5	124.1	124.4	125.0	124.60	···
118	124.5	124.7	124.6	124.6	124.7	124.62	···
119	125.3	124.2	125.2	124.6	124.3	124.72	···
120	125.6	124.3	124.4	124.7	124.9	124.78	···
121	124.9	124.6	125.0	124.3	124.2	124.60	···
122	124.9	124.3	124.5	124.1	124.6	124.48	···
123	124.2	124.0	124.5	124.6	124.3	**124.32**	**11**
124	125.4	125.5	124.5	124.5	124.2	124.82	···
125	124.1	124.1	126.1	124.3	124.2	124.56	···
126	124.1	125.9	124.1	124.7	125.3	124.82	···
127	124.1	124.3	124.1	124.2	124.5	**124.24**	**4**
128	124.2	124.2	124.4	126.3	124.5	124.72	···
129	124.2	124.7	124.6	124.3	125.1	124.58	···
130	126.5	124.1	125.3	124.3	124.4	124.92	···

Table 5: 30 samples of size $n = 5$ corresponding to a 30 hours sequence of production

the value of $\bar{X}_i$ for $i = 101, \ldots, 111$ also verify that $LCL < \bar{X}_i < UCL$. Then the 112th sample ($\bar{X}_{112} = 124.32$) is below $LCL = 124.37$ of the SWV-$\bar{X}/S$ sub chart. This implies that $CRL_1 = 112 > L = 9$ and we can conclude that, up to this point, the process seems to be perfectly in-control. From the 113th sample to the 122nd sample, the values of $\bar{X}_i$ verify that $LCL < \bar{X}_i < UCL$. The 123rd sample ($\bar{X}_{123} = 124.32$) is again below $LCL = 124.37$. This implies that $CRL_2 = 123 - 112 = 11 > L = 9$ and we can conclude that up to this point, the process is still in-control. Then samples 124, 125 and 126 show that $LCL < \bar{X}_i < UCL$, but sample 127 ($\bar{X}_{127} = 124.24$) is below $LCL = 124.37$. Thus, we have $CRL_3 = 127 - 123 = 4 < L = 9$ and we can conclude that an out-of-control situation occured corresponding to a downward shift in the process mean (i.e. less yogurt in each of the cups), probably due to a clog in the pipe used for filling the cups.

5 Conclusions

A synthetic SWV-$\bar{X}$ chart for skewed populations is suggested in this paper. The ARL results have shown that the synthetic SWV-$\bar{X}$ chart gives a more favourable performance than the synthetic WV-$\bar{X}$ chart when the mean of an underlying process from a skewed population shifts downward or in the negative direction. Consequently, the synthetic SWV-$\bar{X}$ chart can be a favourable substitute for the synthetic WV-$\bar{X}$ chart in process monitoring when the mean of a skewed population is likely to shift downward, whenever a change occurs.

Appendix

Let us focus on transformations of form $Z = a + bg(Y)$ of the random variable Y, where a and $b > 0$ are two parameters, where g is a monotone increasing function, and where Z is a $N(0,1)$ random variable. It is very easy to show that the cumulative distribution function of the random variable Y is:

$$F_Y(y) = \Phi(a + bg(y)).$$

If c and $d > 0$ are two additional parameters such that $Y = \frac{X-c}{d}$, then we can

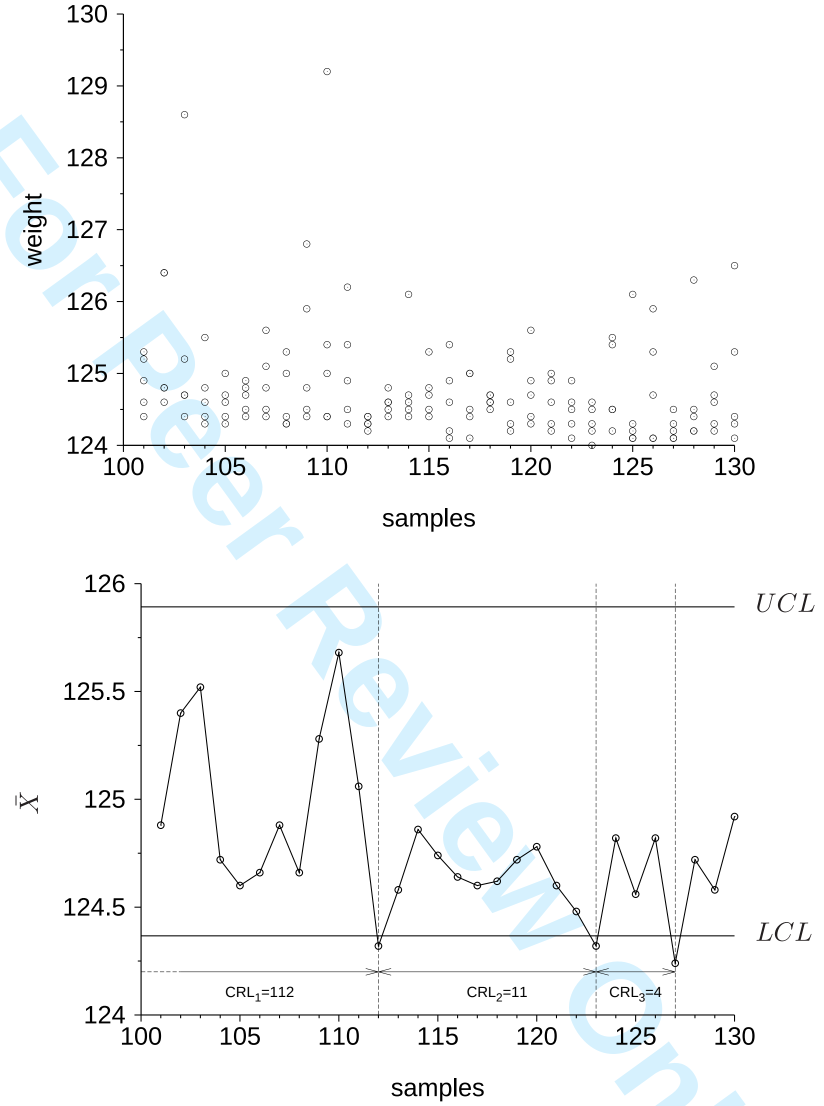

Figure 3: (top) 30 samples of size $n = 5$ corresponding to a 30 hours sequence of production, (bottom) the corresponding Synthetic SWV-$\bar{X}$ chart

straightforwardly deduce the cumulative distribution function of the random variable X, i.e., $F_X(x) = F_Y(\frac{x-c}{d})$. There is a large number of possibilities for choosing an adequate function g. Johnson (1949) has proposed a very popular system of distributions based on a set of three different functions:

- $g_L(Y) = \ln(Y)$ and $d = 1$. The distributions defined by this function, called Johnson S_L (lognormal) distributions, are defined on $[c, +\infty)$ and the cumulative distribution function is equal to:

$$F_L(x|a, b, c) = \Phi(a + b\ln(x - c)).$$

- $g_B(Y) = \ln(\frac{Y}{1-Y})$. The distributions defined by this function, called Johnson S_B distributions, are defined on $[c, c+d]$ and the cumulative distribution function is equal to:

$$F_{JB}(x|a, b, c, d) = \Phi\left(a + b\ln\left(\frac{x-c}{c+d-x}\right)\right).$$

- $g_U(Y) = \ln(Y^2 + \sqrt{Y^2+1}) = \sinh^{-1}(Y)$. The distributions defined by this function, called Johnson S_U distributions, are defined on $(-\infty, +\infty)$ and the cumulative distribution function is equal to:

$$F_{JU}(x|a, b, c, d) = \Phi\left(a + b\sinh^{-1}\left(\frac{x-c}{d}\right)\right).$$

Let μ_2, μ_3 and μ_4 denote the 2nd, 3rd and 4th central moments, respectively, of the random variable X. Johnson has proven in his paper that (a) for every skewness coefficient $\beta = \mu_3/\mu_2^{3/2}$ and every kurtosis coefficient $\psi = \mu_4/\mu_2^2 - 3$ such that $\psi \geq \beta^2 - 2$ there is one and only one Johnson distribution, (b) the S_B and S_U distributions occupy non-overlapping regions covering the whole of the skewness-kurtosis plane, and the S_L distributions are the transitional distributions separating them.

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